

Simple Inventory System

Setting

A company is seeking to manage the inventory level for a single product. Customers purchasing the product arrive according to an arrival process $\{t_A\}$. Each arriving customer attempts to purchase a random number of items, D . The inventory level at the company is reviewed periodically and a decision is made whether or not to place an order from its supplier. When the company places an order, it takes a certain amount of time (“lead time”) for the order to arrive. Due to a variety of circumstances, the lead times are random. The company uses a $\langle s, S \rangle$ policy for its ordering decisions. If the inventory position at review time is below s , then an order is placed that is the difference between it and the number S . The inventory position includes the amount of the product on-hand and the amount of the product that is on-order (that is, has been ordered from the supplier but not yet received).

When a customer’s order cannot be filled, the unfilled portion is put on backorder. When the company receives a shipment from its supplier, backorders are immediately filled and the remainder put in stock. For example, if there are 5 items in stock and a customer wants to buy 8 items, the customer is given the five items in stock and the remaining three are backordered.

Extensions

1. *Adding costs.* Every time an order is placed with the supplier, it costs the company a fixed amount, K , plus C per item ordered which is incurred when the order is received. Each item that is held in stock at the company incurs a “holding cost” of $\$h$ per unit time, which starts as soon as the item is placed in inventory and ends when the item is sold. Each item that is on backorder incurs a shortage cost of $\$ \pi$ per unit time, which starts as soon as the item is backordered and ends when the backorder is fulfilled.
2. *Perishable items.* After a certain amount of time on the shelf, items spoil and must be discarded. An item is not discovered to have spoiled until it is taken from inventory for a customer. In general, different items will have different shelf lives.
3. *Fixed Batch sizes.* All orders from the supplier must be purchased in batches of size B . Each order therefore, when placed after a review, is of size nB , where n is the largest integer such that $nB \leq S$.
4. *Fixed Order Quantity.* Instead of ordering “up to” S , each time an order is placed it is a fixed quantity Q . That is, when inventory is reviewed, if the inventory position is less than r , then an order for an amount Q is placed.